Could Shakespeare have calculated the odds in Hamlet's wager?

Sam C. Saunders

> *Osric:* *The King, Sir, hath wager'd with him six Barbary horses: against the which he has impawned [staked], as I take it, six French rapiers and poniards, With their assigns, as girdles, hangers and so: three of the carriages, in faith, are very dear to fancy, very responsive to the hilts, most delicate carriages, and of very liberal conceit.*
>
> *Hamlet:* Act V, Scene 2

EACH of us reads, understands and appreciates Shakespeare through our own filter of intelligence, education and experience. Thus lawyers and doctors are quick to identify those passages that exhibit arcane knowledge of law and medicine. Specialized information in natural science and classical literature can be found by experts in those fields, while travellers mention geographical details which could only have been acquired through European travel. Such erudition seems far more likely for Edward de Vere than for William of Stratford, and such comparisons constitute one of the most convincing arguments for the Oxford hypothesis. Yet any indication in the plays of mathematical acumen is rare.

One of the most contentious passages in the canon is the wager made by Claudius in his effort to eliminate Hamlet by means of a deadly duel with Laertes, a passage that, after 400 years of discussion, still lacks proper exegesis. It is our hope that an analysis of the mathematics involved in setting the odds may offer some help. After surveying some of the commentaries and explanations that have been offered on this point since the 1750s, we hope to show how the odds quoted for the duel might have been determined using a sixteenth-century knowledge of probability, an hypothesis that, we believe, provides a logical and consistent interpretation of Shakespeare's dialogue relating to the match.

The Wager

In Act V Scene 2, Claudius wagers that Hamlet will win in a passage of arms with Laertes, under certain conditions. To ensure Hamlet's participation, he places a valuable stake, at odds, on Hamlet's skill, sending the foppish courtier Oscric to inform the prince: "My Lord, his Majesty bade me signify to you, that he hath laid a great wager on your head. . . . "

Hamlet reacts to the stated magnitude of the wager (above):

> Hamlet . . . But, on: six Barbary horses against six French swords, their assigns, and three liberal-conceited carriages; that's the French bet against the Danish?
>
> Osric The King, Sir, hath laid, that in a dozen passes between yourself and him, he shall not exceed you by three hits: he hath laid on twelve for nine; and it would come to immediate trial . . . .
>
> Hamlet Let the foils be brought . . . and the King hold his purpose, I will win. For him if I can; if not I will gain nothing but my shame and the odd hits.

J.A. Kilby found this dialogue "Perhaps the most puzzling in all English literature" (8). And in his 1765 edition of *Hamlet*, Samuel Johnson said:

> This I do not understand. In a dozen passes one must exceed the other more or less than three hits. Nor can I comprehend how, in a dozen, there can be twelve to nine. The passage is of no importance; it is sufficient there was a wager. (432)

A somewhat later commentary "by the most eminent Alexander Chalmers A.M." amplifies this confusion. We quote from an 1805, privately printed, nine-volume edition of Shakespeare's plays, employing the corrected text of George Steevens:

> As three or four complete pages would scarcely hold the remarks already printed, together with those which have lately been communicated to me on this very unimportant passage, I shall avoid both partiality and tediousness, by the omission of them all. I therefore leave the conditions of this wager to be adjusted by the members of Brookes', or the Jockey-Club at Newmarket, who on such subjects may prove the most enlightened commentators, and most successfully bestir themselves in the cold unpoetick dabble of calculation. (274)

Chalmers' sage advice on the use of numerate (mathematically literate) commentators has not always been subsequently heeded.

Some editions avoid any commentary about the stipulations of the wager. The didactic Yale Shakespeare edition (1966) says only "The exact details of this wager are a matter of doubt" (200). Others, including the voluble and numerate Isaac Asimov, are silent. But some are argumentative. From a reissue of the 1885 edition of *The Valorium* we have the following response:

> Tschischwitz assumes that "a dozen" is merely an indefinite number, and gives an elaborate calculation on the basis of twenty-one [9+12] rounds. (It may be said of all these calculations what Clarendon said of one of them, they are doubtless correct, but do not explain the form in which the wager is put). (433)

One numerate commentator "who thought that a mathematical approach would be most promising" and who did bestir himself with calculation was Evert Sprinchorn, who: 1) construed "laid twelve for nine" to mean the odds were 4 to 3 against Laertes; 2) showed these odds were very different from those calculated by the commonly assumed 12-bout rules for winning the match; and 3) proposed an interpretation of rules specifying the winner as the first to obtain 3 consecutive hits. He then calculated the resulting odds when both contestants are equally skilled, showing them to be very close to those specified; and 4) argued that his rule interpretation both explained the accompanying dialogue and increased the drama of the duel scene by providing the contestants with a realization of the games impending end.

Springchorn's first article, published in the *Columbia University Forum* in 1964, was so well received it was chosen for inclusion in the prestigious publication by Zinter in 1966. After amendments by mathematical referees, it was published again in 1970 in the also prestigious *American Statistician*. Sprinchorn had supposed that a misinterpretation of the rule, due to its ambiguity or unfamiliarity, was far more likely to have occurred than a large arithmetical discrepancy between the actual odds and those set in the wager.

Clearly, if a reader does not frame any rule, or cannot calculate the odds entailed by his assumption, then a numerical discrepancy, of any magnitude, would go unnoticed. But the innumerate Shakespearean scholars were not convinced by Sprinchorn's interpretation, as seen in both in the cleverly-phrased and condescending, but nugatory, rebuttal in 1968 by Kilby in *Notes and Queries* and later by Jenkins, editor of the 1982 Arden Shakespeare (561-3), who begins by quoting Osric: "The King, Sir, hath laid, Sir, that in a dozen passes . . . , etc."

> The terms of the wager pose an insoluable problem. The first is [that] the King bets that in a match consisting of twelve passes or bouts, Laertes will not make three more hits than Hamlet. Johnson's objection to this [interpretation that the match requires 12 passes] is "In a dozen passes one must exceed the other by more or less than three hits," though more cited is not valid; for "more or less than three" will be decisive, and "passes" are not the same as "hits" . . . though not all commentators agree that "Nothing neither way" implies that the bout is at an end. The difficulty, however, comes when the second statement is added: he hath laid on twelve for nine. This is most naturally taken to refer to the excess of three hits—twelve for Laertes for nine by Hamlet, which

---

Sam Saunders retired as a Professor of Mathematics/Statistics at Washington State University in 1997, where he had served for a quarter of a century. Before that he was a Research Specialist at the Boeing Scientific Re-search Laboratories for sixteen years and had taught at the University of Washington for twenty years. He was employed for twenty years by the NRC as a consultant to The Advisory Committee on Nuclear Safeguards, for ten years by Mathematical Analysis Research Corporation, for twenty years by Scientific Consulting Services and for ten years by the National Institute for Standards and Technology. He has studied Latin, Literature, Physics, and Mathematics, is a member of Phi Beta Kappa, and a Fellow of the American Statistical Association. He has been interested in Shakespeare since he read Charles and Mary Lamb's Tales from Shake-speare *when he was in grade school.*

> is not compatible with "a dozen passes." Aside from this . . . there is an ambiguity in the pronoun, he. . . it is natural to presume that the subject is the same. But . . . if you bet on a man's not exceeding his opponent by three hits you cannot lay on but are laying against his making twelve for nine. In either event it certainly looks as through the two statements were meant to correspond. . . . More commonly there have been attempts to find an interpretation of one or both of the statements which would permit them to be reconciled. The precise terms of the first leave little room for maneuver. Not all the mathematical bravura of Sprinchorn for computing "The Odds on Hamlet" can possibly persuade us in defiance of plain English that "to exceed you three hits" means to score three hits in succession. . . . / [I]t is hard to see how odds of "twelve to nine in favor of Laertes" (as Sprinchorn did) can apply to a wager in which the odds in one sense are set at `six Barbary horses against six French swords' and in another at "three hits" (561-3).

Even today agreement has failed on this particular point. On February 6, 2004 an attempt was made in the London Times Literary Supplement by six English literati, headed by the Shakespeare scholar Frank Kermode. In a headline-featured article entitled "Odds on The Hamlet Duel" they gave their differing personal opinions on the odds. Their odds, based on subjective opinion (except for Kermode, who quoted Sprinchorn) when calculated, ranged from 14 to 1 for Hamlet up to 5 to 1 against him—and this group contained a gambler and a fencing expert!

Let us list here what we regard as misinterpretations hithertofore:

- *The terms . . . pose an insoluable problem*: No, they are only incomplete. Does it not seem more reasonable that Shakespeare would omit a clear specification of the rules for a fencing engagement before a sixteenth-century audience very familiar with such proceedings than make a gross miscalculation of the odds following therefrom?

- *Passes are not the same as hits*: Indeed not, thus: "Nothing neither way" (5.2.305) indicates no hit. But then, and now, all bouts continue until a hit is made. *Bouts* were commonly referred to as *passes* since at that time each pass usually resulted in a hit. All attacks in the sixteenth century would have been by thrusts and passes in the "old style": "Hamlet: Come on Sir. Laertes: Come, My Lord." (5.2. 277-8), rather than by either the sudden lunge or fléche developed later. There was then, during swordplay, much more grappling, scuffling and grabbing of opponent's blades, so the accidental exchange of swords between contestants, as in 5.2.306, would not have seemed unlikely. Appreciation for this scene is increased by familiarity with sixteenth-century dueling practice, uncommon today among audiences familiar only with choreographed swordplay as seen in films.

- *He hath laid on twelve for nine*: This does not refer to the excess of 3 hits but, in accord with the plain English of Hamlet, as in 5.2.164, where "He" refers to the King. So it means Claudius's wager exceeds that of Laertes in proportionate values of 12 to 9!

- *"If you bet on a man's not exceeding . . . you cannot lay on . . .":* Of course one can lay odds on any event whether its specification in English contains a negative or not. (Harold Jenkins is not as absolute in his language as was Hamlet's gravedigger.)

- *How odds of "12 to 9 in favor of Laertes" can apply to a wager when the odds are . . . given as both horses against swords and as 3 hits*: This comment proves its author did not read Sprinchorn or did not understand him. The odds "12 to 9 in favor of Laertes" were what Sprinchorn falsifies by showing the odds are very nearly 12 to 9 against Laertes.

Odds long antedated probability as an expression of likelihood in wagers. The laying of odds in common usage has often the same meaning as setting the handicap. Confusion also arises because the bookmaker's convention for accepting bets was, and is, always to quote first the odds against, i.e., the odds on a horse to win, or on the outcome of any event for that matter, being 6 to 1 means one can bet \$1 and (possibly) win \$6. (This is what the members of the Jockey Club at Newmarket would surely have maintained when the equitable probability of the horse winning is set at $1/(6+1) = 1/7$.) But to suppose that this convention was used by Claudius leads to an immediate contradiction. Today, *odds* are more commonly used in wagers while *probabilities* are used in calculations.

What did Shakespeare know about swordplay?

Shakespeare seemed to be well-informed about fencing. The zenith of the dueling craze reached England during the sixteenth century, several decades later than in France and Italy, where it followed soon after the introduction of the rapier c.1450. In fact, tournaments using the sword and buckler were still being held in Elizabethan England, where the argument concerning the efficacy of edge versus point continued for almost five decades until both were made obsolete by firearms. Elizabeth's enthusiasm for Italian culture was shared by the upper classes, and gentlemen of fashion succumbed by wearing both Italian ruffs and the long rapier. The attractiveness of this ensemble was due not only to its stylishness but also to its perceived capacity for menace at a time when an exaggerated sensitivity to offense was affected by all "men of honor"—"why thou will quarrel with a man that hath a hair more or less in his beard than thou hast" (*Romeo and Juliet* 3.1.17-18)—and Touchstone too, famously, ridicules the sensitivity to slight necessary for offense to be taken and a formal challenge issued: "the retort discourteous . . . the quip modest . . . the reply churlish . . . the lie direct" (*As You Like It* 5.4.68-83).

There is no record in England of a duel fought by illustrious men "requiring satisfaction" before the sixteenth century (Cohen 49), very few occurred before the reign of James I. But in 1579 there was the well-known quarrel on the royal tennis court that led to a challenge between Edward de Vere and Philip Sidney. Sidney, it seems, was a fencing enthusiast who once wrote to his brother

urging him to "practice the single sword and then with the dagger" and to "let no day pass without an hour or two of exercise" (32), as presumably he did himself. The duel was prevented when the Queen intervened, as she always did whenever threats of this kind arose among her courtiers.

Until Shakespeare, no playwright had used the swordfight so well and so importantly in a plot. The word *sword* is referenced 437 times in the canon and swordfights in the plays are always at stage center: Antony versus Octavian, Iago versus Casio—there is even a "trial by combat" in Henry VI. In fact Shakespeare gave rise to an entire profession of stage fight choreography that might never have occurred without him. That he knew rapier play well is shown in *Romeo and Juliet* (2.1.173-78) where Mercutio's opponent is Tybalt, an obvious reference to one Gerard Thibault, a famous London fencing master who emphasized metronomic timing, later publishing a tome on the subject: *Academie de l'éspée*. Mercutio derides Tybalt as a "villain who fights by the book of arithmetic. . . . He fights as you sing prick-song—keeps time, distance and proportion. He rests his minim rests—one, two and a third at your bosom" (2.3.21 et seq). Moreover Hamlet's question, "These foils have all a [single] length?" reflects the precaution every good fencer should take before a critical match, not his concern that Laertes was being given an advantage.

The duel interpretation

Bent on Hamlet's assassination, Claudius wins Laertes to his plot by convincing him that it's his duty to avenge his father's murder and his sister's suicide. He entices Hamlet by making the wager sufficiently valuable and flattering that Hamlet feels obligated to compete. He disguises his purpose by giving odds favoring Hamlet merely by placing a stake proportionately more valuable than the French stake.

The point of principal confusion in Hamlet's duel scene is the interpretation of the conditions under which Claudius wins the wager: "in a dozen passes between the two, [Laertes] shall not exceed [Hamlet] by three hits." No further specifications are given. Consider these three alternative interpretations: Claudius loses (and Laertes wins) if, during a match of 12 passes:

Rule I: Laertes' total hits exceed Hamlet's by at least 3.

Rule II: Laertes is the first to make 3 successive hits.

Rule III: Laertes is the first to achieve a total of 3 more hits than his opponent.

But these rules do not delineate the conditions for Hamlet to win. We assert these rules to mean, respectively:

Rule I: Hamlet wins if he makes 5 hits before Laertes makes 8 hits;

Rule II: Whoever first makes three successive hits wins and if no one does, in 12 passes, Hamlet is the winner;

Rule III: The match is won by the player who first obtains an excess of 3 hits more than his opponent. If this "excess of 3" does not occur in 12 passes, Hamlet wins by default.

Sprinchorn considered only the first two interpretations as reasonable. He shows that the most commonly made interpretation, *Rule I*, is incommensurate with the odds given when both players are equally skilled, since the probability of Laertes's success is then 397⁄2048 = 0.194. Consequently the correct odds-for Laertes should be only 1 for 4 (shameful odds for a prince) not at all close to the 3 for 4 given. Sprinchorn advocates *Rule II* because his calculation of the probability of Laertes winning this game, assuming both fencers are of equal skill, is 1815⁄4096 = 0.443. This value is very close to 3⁄7 = 0.429, the equitable probability from the given odds-for Laertes of 3 to 4.

Sprinchorn's paper was labeled silly mathematical bravura by the Stratfordians for having interpreted II as the winning condition "in defiance of plain English" (see Kilby and Jenkins). While *Rule II* might not seem obvious, it does make the third bout in *Hamlet* decisive and so heightens dramatic tension in the scene, since if Hamlet scores the first three hits, Laertes will lose the match, thus failing to fulfill either his agreement with Claudius or his revenge against Hamlet. However, under *Rule III* the dialogue takes on the same meaning as regards the first 3 hits. After the second touch by Hamlet the King says: "Our son shall win." Under *Rule I*, this would be premature since the match could not be decided, at the earliest, until the fifth bout and, at the latest, until the twelfth bout. But in a match that can be won by making 3 consecutive touches, or by being ahead by 3 wins (as under *Rules II* or *III*), when, as in their duel, 2 have already been won, winning becomes a reasonable hope and expectation. Either rule adds dramatic interest to the third passage of arms during the duel scene. As explained by Sprinchorn in 1970, Shakespeare has masterfully intensified the drama by placing Laertes in a desperate situation after the second hit, that is, if one goes by either *Rule II* or *III*.

Most persons who have considered it have concluded that *Rule I* was meant, but their inability to calculate the resulting probability that Hamlet wins, viz., 1651/2048 = .806, when both players are equally able, shields them from the magnitude of the discrepancy between the odds Claudius gave and those he should have given. Some commentators on *Hamlet*, favoring Rule I, have remarked that if Shakespeare had meant *Rule III* he would have used the phrase "at any time during" rather than just "in." (On the other hand, if Shakespeare meant *Rule I*, he should have said "after having finished" rather than just "in.")

We maintain that *Rule III* is the correct interpretation. First: it is a common game at fence between persons of unknown relative ability (see Cohen), since it forestalls embarrassing scores between persons of vastly different skill. Second: the consequent odds are very near to those wagered and are as close as empirical determination could have made them. Moreover, it too makes the third pass the climax of the drama of the duel.

Shakespeare's attention to detail in his plays is so well known that discrepant odds in *Hamlet* become the principal objection to *Rule I* among those who can do the probability calculation. Numerate persons can accept an ambiguity or a small anomaly in the dialogue far more readily than they can accept a large discrepancy between the proffered odds and the actual ones.

The Sprinchorn calculations

If the odds determined by the wager are 12 to 9 in favor of Laertes, then the equitable probability that he will win is 12/(12+9) = 4/7. Therefore the equitable probability that Hamlet will win is 3/7. Inexplicably Sprinchorn rejects this, saying:

> [R]eaders might wonder if the odds against Hamlet were equalized in the amount wagered rather than in the terms of the duel . . . . But such an inference is not supported by anything else in the play and is contradicted by the fact that Laertes accepts the three hit handicap. We must assume that Claudius and Laertes regard the horses as equal to the swords in this gentlemen's wager. (*Statistician 14-17*)

It is qualitative nonsense for any bettor to say that odds have been given when the wagers on both sides are of equal value. Sprinchorn was a drama professor at Vassar; however, his calculations are correct and his reasons why the dramatic interpretation requires that the duel be over if Hamlet wins the first three passes are very persuasive.

Under Rule I: Given that both players are equally skilled at fence, one may calculate the outcome as being identical to flipping a fair coin marked *H* or *L* twelve times. The probability that *L* will occur at least 8 times in 12 trials is:

$$\frac{\binom{12}{8}+\binom{12}{9}+\binom{12}{10}+\binom{12}{11}+\binom{12}{12}}{2^{12}} = \frac{494+220+66+12+1}{4096} = \frac{794}{4096} \doteq 0.19385. \qquad (1)$$

Under *Rule II*: How many of the $2^{12} = 4096$ possible outcomes will have neither *H* nor *L* occurring 3 times successively during the 12 trials? The answer is, by somewhat more difficult com-

$$2\left[\binom{12}{0}+\binom{11}{1}+\binom{10}{2}+\binom{9}{3}+\binom{8}{4}+\binom{7}{5}+\binom{6}{6}\right] = 466. \qquad (2)$$

binatorial analysis (see Saunders or Sprinchorn, *Statistician*) is:

Then by symmetry, of the possible trials remaining, 4096 – 466 = 3630, half, viz., 1815, have a first triplet of consecutive *L*s. Hence the probability of Laertes winning is 1815/4096 = 0.443 and Hamlet's probability of winning the wager is 2281/4096 = 0.557.

On this "very unimportant passage," I too shall avoid partiality and tediousness, and only maintain that what the plain English in the scene means is that, although Laertes is reputedly the superior fencer, the odds given by Claudius for Hamlet to win the match are nonetheless 4 to 3, and

"Laertes to exceed Hamlet by 3 hits in 12" means that the match is won by Laertes only if he gains an excess of 3 total hits over Hamlet before Hamlet has done the reverse during a match of 12 bouts.

Differences among these three interpretations can be seen in this sequence of outcomes of 11 passes, (H,H,L,H,H,L,H,H,L,L,L). Under *Rule II*, Laertes wins on the 11th bout (he gets 3 hits in a row) despite Hamlet having scored 6 times to his 5. Under *Rule I*, Hamlet would have won on the 7th bout (having gained 5 hits before Laertes has gained 8) while under *Rule III*, Hamlet would have won on the 5th bout (being ahead by 3 hits). Other sequences can be equally instructive.

Why was the game not clearly defined by Shakespeare? Surely not, as Samuel Johnson held, because it was a minor dramatic point, but because at the time that it was written such fencing matches and their etiquette were commonplace, so no further explanation was needed. There were various rules then for how a duel was to be terminated, depending upon its seriousness: 1) "satisfaction required" usually meant it would be ended only by an incapacitating wound; 2) for matched players at fence, the end came after one obtained, say, the tenth touch, or when the one ahead in touches reached a fixed time limit, whichever came first; or 3) for players of unknown relative ability, the match was often ended by an excess-of-3 touches, thus avoiding the embarrassment of one player suffering an 11-1 or a 9-3 defeat in a match fixed at 12 bouts.

The cold unpoetick dabble of calculation

We now show that *Rule III* implies a favorable wager even were Hamlet not quite as skillful as Laertes. The members of the Jockey Club would surely have said that "laid on 12 for 9" denoted the actual, not just the proportionate value, of the respective wagers (presumably in ducats, the coin then in use in Elsinore, see 4.4.25). Claudius laid a wager worth 12 ducats and Laertes one worth 9 ducats, but it's likely that Shakespeare reckoned the value of the stakes in terms of the Elizabethan sovereign or pound (pound coins were first issued in 1583). If so, the value of Claudius's stake of 6 Barbary horses was worth £12 while the value of Laertes stake of 6 French sword-sets (rapier and poniard with their belts and scabbards) was £9. This was not an insignificant sum, since in Elizabethan times £1 was worth approximately \$680 today.[1] So the value of the wager was about \$8160 for Claudius against \$6120 for Laertes. The prices of French sword pairs, to be commensurate, would have averaged one and a half pounds (£1 10s). That this is the approximate value for "six rapiers and poniards with their assigns" is confirmed by the Earl of Oxford having paid £1 6s 8d for a rapier, dagger and belt c.1570 (Anderson 40). Moreover, two centuries later, in 1762, James Boswell recorded that he paid 5 guineas for a silver-hilted sword, sheath and belt made by George III's own sword cutter (63).[2]

If h denotes the probability that Hamlet wins the match, then Claudius receives the French swords and assigns, worth £9, while if Hamlet loses the match, which will occur with probability $1-h$, then Claudius loses the Barbary Steeds, worth £12. The wager is favorable to Claudius, and hence unfavorable to Laertes only if his (Claudius's) expected gain is positive. The expected gain for the wager then is $9h-12(1-h)$, which is positive only when $h > 4/7$. Thus, under any rule, Claudius has positive expected gain when Hamlet's probability of winning the match exceeds 4/7.

Did Shakespeare know the probability of Claudius winning this wager could exceed 4/7 when on each pass Hamlet's chance of scoring a hit is slightly less than Laertes? Such a situation is consistent with Laertes's reputation as a swordsman without peer, as we and Hamlet are informed by Osric: "You are not ignorant of what excellence Laertes is . . . . I mean Sir, for his weapon; but in the imputation laid on him by them, in his meed he's unfellowed." Later Hamlet himself, if only with the modesty of courtesy, allows that Laertes is the more skilled: "I'll be your foil Laertes. In mine ignorance your skill shall like a star i' th' darkest night stick fiery off indeed."

Horatio reveals his opinion of Laertes's skill when he tells Hamlet:

> Horatio You will lose this wager, My Lord.
>
> Hamlet I do not think so; since he went into France, I have been in continual practice; I shall win at the odds.

By "at the odds," Hamlet means that, despite Laertes's skill, his handicap will even the playing field, requiring as it does, that Laertes score 3 more hits than Hamlet before Hamlet can do the same to Laertes in 12 passes. Presumably Hamlet knows this will be difficult for his opponent since he himself has been "in continual practice" ever since Laertes left for France. Nevertheless, however confident he shows himself to Horatio in private, Hamlet speaks modestly to Claudius:

> Hamlet Your Grace hath laid the odds o' th' weaker side.
>
> King I do not fear it; I have seen you both: But since he is bettered, we have therefore odds.

This discourse is consistent with the King's expectation of winning, because, knowing no doubt of Hamlet's practise, he believes his nephew's skill is not so much less than Laertes as to undercut Laertes's handicap. But the King's £12 wager on Prince Hamlet against £9 on Laertes ("twelve for nine"), is also a manifestation of his sense of *noblesse oblige*. (The phrase "is bettered" refers to Laertes's inferior social status, not to his superior swordsmanship or his handicap. Recall Hamlet's outcry when he realizes that he's killed, not the King, but his minister: "Thou wretched, rash, intruding fool, farewell! I took thee for thy better" [3.4.31-2]). Thus Claudius gives odds of 4 to 3 on Hamlet, which to an innumerate Laertes seems more than fair since he believes himself to be the better fencer. But how much lower can Hamlet's skill be for the odds to remain in Claudius's favor? The difference in probabilities of success between Hamlet and Laertes under all three rules can be seen in Figure 1 (page 13) as a function of the probability that Hamlet wins each pass.

The counter-intuitive knowledge that this wager is favorable to Claudius must have been based either on Shakespeare's arcane understanding of the true odds for such a game or upon the amazing coincidence that he just happened to create such odds by accident. Recall Claudius's order: "If Hamlet give the first or second hit or 'quit [requite] in answer of the third exchange, let all the battlements their ordinance fire!" (5.2.268-70). Why celebrate after so few hits? Claudius is demonstrating to the assembly that his sole concern is with the wager, this in anticipation of any possible suspicion following what he is certain will be Hamlet's imminent demise. If the contest were proceeding under *Rule I*, it would have to continue until at least five passes were completed, even if Hamlet won them all, thus giving no cause for such an early celebration. However, if they are proceeding as per *Rule II* or *III*, if the more skilled Laertes wins all three of the first passes, Claudius will lose, so he dissembles by calling for a celebration that he believes will not take place.

In addition, when earlier, Hamlet having scored his second hit without Laertes having scored at all, the King's remark to Gertrude, "Our son shall win," shows clearly that he thinks the match will be terminated if Hamlet wins the third round, which happens under *Rules II* or *III*. Finally, it can only be under *Rules II* or *III*, that, Hamlet having won the first two passes, Laertes realizes that if Hamlet wins the third pass as well, he will lose the match and therefore the opportunity to "incidently" cause Hamlet's death. It is Laertes's realization that the match may terminate without ever offering him an opportunity to prick Hamlet with the poisoned sword that leads to his dastardly direct attack and so, inexorably, to the deaths of all involved, something that would not occur under *Rule I*. Thus it seems a most likely interpretation of the rules under which the match was played is that of *Rule III*.

Also note that a calculation of Hamlets's probability of winning so as to determine the correct odds would, under Rules I or II, have been impossible during the sixteenth century, even under the assumption of equal skill (Saunders 12). This is because the combinatorial coefficients as shown in equations 1) and 2) are necessary, and such knowledge lay a half-century in the future with their discovery by the French philosopher/probabilist, Blaise Pascal. Under *Rule III*, however, where equal skills are assumed there is an elementary calculation that can be made to obtain the correct answer, based on information in a book that was available at the time, the *Liber de Ludo Aleae* (*Book on Games of Chance*).[3] Its author was the foremost expert on gambling and games, the mathematician/physician Jerome Cardan. But would Shakespeare have known Cardan's work?

De Vere and Cardan

In 1573, when he was twenty-three years of age, the Earl of Oxford sponsored the publication of *Cardanus Comforte*, a translation from Latin into English of *De Consolatione*, a book of moral aphorisms published in Venice in 1542. This is the book long thought by certain scholars to have been in Hamlet's hand in Act II Scene 2 when, in baiting Polonius, he uses tropes and language that seem to derive from Cardan's book.

Girolamo Cardano (1501–1576) of Milan was a scholar of the Italian Renaissance, who excelled in the fields of mathematics, medicine, philosophy and astrology for forty years while also gambling regularly at chess and with dice, casting horoscopes, and writing poetry, and who, during the Counter Reformation, was arrested for heresy by the Inquisition. (His book on science and natural philosophy, *De Subtilitate Rerum*, had been placed on the Index.) After his examination (likely conducted similarly to that of Galileo) he was placed under surveillance (house arrest) until his death in 1576 (Ore). Cardan's imprisonment on intellectual grounds occurred two decades before the arrest and execution of Giordano Bruno and a half-century before that of Galileo. Overall Cardan left 131 printed works and 111 unprinted manuscripts in philosophy, medicine, science and mathematics. He also wrote one of the first self-analytical autobiographies in the West, which,

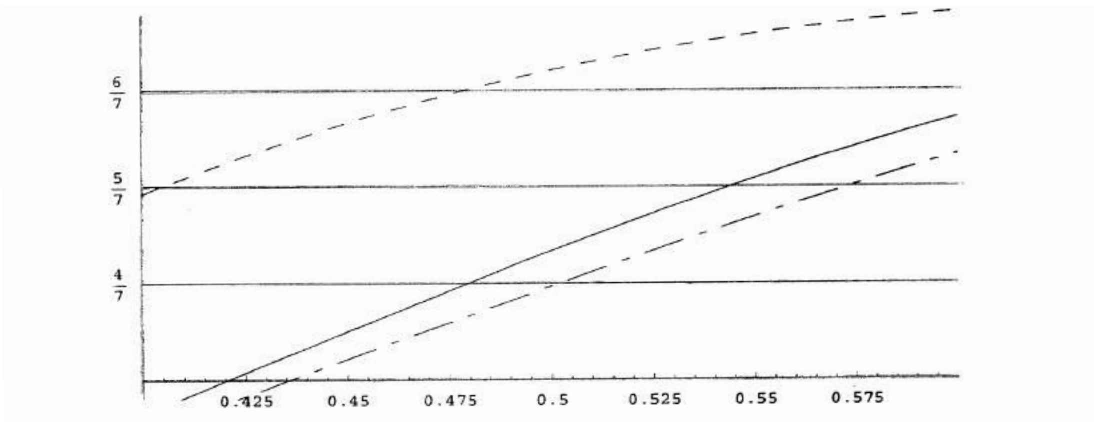

Figure 1: Hamlet's Probability of Success from top to bottom, I,III,II

amazingly, is still in print (Stober). Though largely unrecognized today, Cardano was one of the most remarkable characters of the Renaissance.

"Primero" was Cardan's favorite card game; in *Ludo Aleae* he devotes more space to it than all other card games combined. Shakespeare mentioned this game in *Merry Wives* and *Henry VIII* and made allusions to it in several other plays at a time when there existed nothing in English that explained the rules.

Some mathematical bravura

Let p and $q = 1- p$ be the respective probabilities that Hamlet and Laertes win each bout. The probability $H(p)$that Hamlet, i.e. Claudius, wins the wager under which one of the three rule interpretations will be denoted by an affix, I, II, or III. Note if $L_{\mathrm{III}}(q)$ is the probability that Laertes wins under *Rule III* then $H_{\mathrm{III}}(p) = 1 - L_{\mathrm{III}}(q)$.

The effect that alteration of the rule has on the probability of Hamlet's winning the wager is given, as a function of p, by the graphs of H_{I} (in the dashed line), H_{II} (in the double dashed line) and H_{III} (in the solid line) which are presented in Figure 1 (above). The exact formulae for $H(p)$, under the three differing rules, are given in Saunders 2006.

For example, Claudius winning under the commonly accepted *Rule I* and under Sprinchorn's *Rule II* are found, respectively, with Hamlet and Laertes being of equal skill, from the appropriate equations to have the respective probabilities:

$$H_{\mathrm{I}}(0.5) = \tfrac{1651}{2048} \doteq 0.81, \quad \text{and} \quad H_{\mathrm{II}}(0.5) = \tfrac{2281}{4096} \doteq 0.56,$$

and these rational numbers agree exactly with those computed by Sprinchorn as in his article in *The American Statistician*.

An ingenious method, used by Saunders to obtain $H_{\mathrm{III}}(p)$ depends upon knowing the probability of a default, i.e., when neither player is ever ahead by an excess of 3 hits in 12 passes. Let D, H, and L be the events, respectively, that a default occurs or that Hamlet or Laertes wins the match outright by scoring 3 more total hits than his opponent. If both players are of equal skill the probabiliites that each player wins outright are equal. Since the sum of these probabilities must be unity, that is Pr[H} +Pr[L] + Pr[D] =1, and Pr[H] = Pr[L], it follows that the probability of Laertes winning the match outright, and the probability of Hamlet either winning outright or by default, are:

$$\Pr[L \text{ wins match}] = \frac{1 - \Pr[\mathrm{D}]}{2}, \qquad \Pr[H \text{ wins match}] = \frac{1 + \Pr[\mathrm{D}]}{2}.$$

One finds under *Rule III* in the general case of unequal skill, *mirabile dictu*, that Pr[D]= $(3pq)^5$ which in the equal-skill case $p = q = 1/2$ becomes $(3/4)^5 = 0.237$. Thus for Hamlet we obtain:

$$H_{\mathrm{III}}(0.5) = \frac{1 + (3/4)^5}{2} = \frac{1267}{2048} \doteq 0.619.$$

It is also possible to calculate numerically the true probability of Hamlet's success on each pass, which gives the equitable probability of 4/7 that Claudius wins. We utilize M*athematica* software and the formulae in Saunders (1960) to find:

$$H_{\mathrm{I}}(0.38) = 4/7, \quad H_{\mathrm{II}}(0.52) = 4/7, \quad \text{and} \quad H_{\mathrm{III}}(0.47) = 4/7.$$

Under *Rule II* or *Rule III*, with Hamlet and Laertes roughly equal in skill, we see the odds set by Shakespeare were almost exact. But only under *Rule III* can Hamlet be not quite up to Laertes's skill level and the odds still hold. This meticulous specificity seems a remarkable achievement in the sixteenth century, confirming once again that "Shakespeare got it right." That this answer could have been guessed without extensive fencing experience and knowledge of Cardano's work in probability, seems virtually impossible. ❦

Notes

1 This value was taken from the bi-monthly historical newspaper, *Old News*, of December-January 2004. Ian Haste, in another determination (unpublished), gives a value of nearly $1000 dollars. The problem of determination of worth is not simple since representative commodities have to be selected for comparison. To compare the price of leather then and now, or the price of diamonds then and now illustrates the problem. Mark Anderson gives a different value: £1 = appx. $275 (75).

2 I have not found any record of the price of Arabian (Barbary) horses during Elizabethan times.

3 Under *Rule III*, divide the match of 12 passes into 4 (fictitious) “innings” consisting of 3 passes each. Letting Hamlet’s win be scored -1 and Laertes win be scored +1 on each pass, the only possible scores for each inning are 3,1,-1,-3 and these occur with respective probabilities 1/8, 3/8, 3/8, 1/8. The probability of no triple win by either player during any of the 4 bouts is $(3/4)^4$ which would produce a sequence of four equally likely ±1s. The probability of not accumulating either three 1s or three -1s in a row during the first three innings is 1 - 2/8 = 3/4. (the fourth inning is finessed) which makes the probability of $(3/4)^5$ for no outright win in the match. Thus Hamlet’s probability of winning the match is $[1+ (3/4)^5]/2 = 0.619 > 4/7$. We compute Claudius’s expected gain to be 9 x .619 – 12 x .381 = .999.

4 A translation is provided by Ore.

Gerard Thibault

Works cited

Anderson, Mark. *"Shakespeare" by Another Name*. New York: Gotham/Penguin, 2005.

Asimov, Isaac. "*Hamlet*." *Asimov's Guide to Shakespeare:* The English Plays. Vol 2. New York: Avenel, 1978.

Boswell, James. *Boswell's London Journal, 1762-63*. Ed. F.A. Pottle. London: Folio Society, 1985.

Brooks, T. and J.R. Crawford, eds. *The Yale Shakespeare*. Revised ed. New Haven: Yale UP, 1966.

Cohen, Richard. *By the Sword: a history of gladiators, musketeers, samurai, swashbucklers and Olympic champions*. New York: Random House, 2002.

De Vere, Edward. *Oxfordian Studies*. Langport: De Vere Society, 2002.

Kilby, J.A. " The Odds on Sprinchorn's 'The Odds on *Hamlet*.'" *Notes and Queries* 212 (1968): 133-6.

Ogburn, Charlton. *The Mysterious William Shakespeare*. (1984). McLean, VA: EPM, 1998.

Ore, Oystein. *Cardano, the Gambling Scholar*. (1953). Princeton: PUP, Chelsea, 1965.

_______________. "Pascal and the Invention of Probability Theory," *American Mathematical Monthly* 47 (1960): 409-419.

Saunders, Sam C., "Did Shakespeare Know the Odds in Hamlet?" submitted for publication in *College Mathematics Journal*, 2006.

Shakespeare, William. "Hamlet." *Arden Shakespeare*. Ed. Harold Jenkins. Walton-on-Thames, Surrey: Nelson, 1997.

_______________. "Hamlet." Ed. Horace H. Furness. *New Variorum*. Vol I. (1885). New York: Dover, 2000.

Sprinchorn, Evert. "The Odds on Hamlet." *Columbia U. Forum* 8 (1964): 41-5; reprinted in *The Practice of Modern Literary Scholarship*. Ed. Sheldon P. Zinter. Glenview, IL: Scott, 1966.

_____________. "The Odds on Hamlet." Reprinted with amendments in *The American Statistician* Dec. (1970): 14-17.

Stober, Jean, trans. and ed. *The Book of My Life, by Girolamo Cardano*. New York: *Review of Books*, 2002.

Stritmatter, Roger. "De Vere's Dedicatory Poem in Cardan's *Comforte* (1573)." *The Oxfordian* 1 (1998): 53-63.

Steevens, George. *The Plays of Shakespeare*. Vol. 9, London: 1805.