

Bayesian Interrogation of the Elizabethan Social Network for First Folio Authorship

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lizabeth I's court favorites and heroes such as Robert Dudley, Walter Raleigh, Francis Drake, and enigmatic polymaths like Francis Bacon and John Dee provided fertile ground for the English Renaissance. Such was the demand for new entertainment in this exciting era that around 2,400 plays were presented from 1590-1642.¹ Of these, about two per cent were the Shakespearian canon.

A playwright's intellectual property was generally protected through registration and censorship approval. However, plays usually had little economic value after the typical performance period of one week. While William Shakespeare may be a notable exception, the vocation of playwright was a hand to mouth existence and often dependent upon the favor of a wealthy patron.

A number of Medici-like patron-families economically sponsored and shepherded players groups. Foremost among them were Robert and Ambrose Dudley, the patrons respectively of Leicester's Men and Warwick's Men. The Stanley family was an early patron of the Lord Admiral's Men and Lord Strange's Men (or Derby's Men), which probably became the Herbert family's Pembroke's Men. Similarly, the de Vere family sponsored a boy troupe, the Oxford's Men and later adopted the Warwick's Men. Royal approval ensued with Elizabeth I's patronage of the Queens' Men, which drew on Robert Dudley's Leicester's Men, and the company with whom William Shakespeare associated, the Lord Chamberlain's Men, evolving into the King's Men under James I's patronage.

Official documents, private letters, insightful commentaries and gossips record Elizabethan social relationships in considerable detail. One of relatively few thin patches in this social fabric is the Shakespeare authorship question. *Prima facie* evidence for William Shakespeare's authorship is indisputable, with that name being

recorded on the latter two-thirds of Shakespearean plays and in the First Folio.

The surfeit of such records rests somewhat incongruously alongside a dearth of independent documentary evidence. This issue may be of little consequence to many persons, but others feel challenged by the mystery of this inconsistency or seek to better appreciate Western culture through developing an improved understanding of Shakespeare's depth of character. The latter group believes that it is a moral imperative to discover whether the dazzling and multidimensional playwright "Shakespeare" was someone other than the sharp businessman, lender and grain hoarder from Stratford-on-Avon portrayed in the few extant legal records.

In recent decades this controversial topic has grown in both intrigue and scope. Over sixty candidates have been put forward for potential authorship and the presentation of arguments is becoming ever more flamboyant. For example, a number of U.S. Supreme Court Justices have expressed opinions about the candidacy of Edward de Vere, 17th Earl of Oxford. At a 1987 moot court, three Justices ruled in favor of the Stratford man. But by 2009 three of Justices favored Oxford's authorship, two favored William Shakespeare of Stratford and four abstained. Edward de Vere's candidacy remains very much alive. A recent feature film, *Anonymous*, with a production cost of \$27.5 million, controversially argued his case.²

The potential authors selected for study are: William Shakespeare of Stratford-on-Avon (1554-1616), Edward de Vere, 17th Earl of Oxford (1550-1604), Christopher Marlowe (1564-1593), Francis Bacon, 1st Viscount St. Albans (1561-1626), Mary Sidney, Countess of Pembroke (1561-1621), Sir Philip Sidney, brother of Mary Sidney (1554-1586), Roger Manners, 5th Earl of Rutland (1576-1612), William Stanley, 6th Earl of Derby (1561-1642), Edward Dyer (1543-1607), Elizabeth I (1533-1603) and Mary Sidney's niece Mary Wroth (1587-1651/3). Mary Wroth is included since she was a renowned poet and author of the first known piece of fiction in the English language. Born in 1587, she would have been only eleven years old when the first plays were printed. Nevertheless, she would be of interest if she contributed to the later plays as part of an authorship group.

The wealth of extant information on Elizabethan social relationships provides a framework amenable to social network analysis. While researchers routinely highlight particular social relationships as *ad hoc* elements in their historical and literature research, formal mathematical social network analysis using random exponential graph models (ERGM) has not hitherto been applied to a more dynamic understanding of important relationships in the Elizabethan theater.

The first part of this research applies new Bayesian ERGM techniques to investigate these eleven authors against the background of the wider Elizabethan Social Network. Over the last five years ERGM techniques have matured using Markov Chain Monte Carlo integration, maximum likelihood estimation and shared partner statistics that address potential model degeneracy.³

The second part applies modern cryptography with log likelihood estimators to a cipher that may increase the authorship probability of Mary Sidney, Countess of Pembroke, who is a prime candidate. The seat of the Earls of Pembroke at Wilton House, near Salisbury, has been a cultural icon for many centuries. Kennedy writes,

“The Earls of Pembroke had from the reign of Henry VIII been encouragers of fine arts, and very early shewed their taste in employing Holbein and Jones in improving their noble seat at Wilton.”⁴

In 1743 Henry Herbert (c.1689-1750), 9th Earl of Pembroke, commissioned Peter Sheemakers to sculpt a statue of William Shakespeare for Wilton House. It was placed in the Black Marble Table Room alongside an ancient bust purchased by his father, Thomas Herbert (1656-1733), 8th Earl of Pembroke, “The Bustos of LYSIAS the Orator, of whom Cicero gives this Commendation: Venustissimus scriptor ac politissimus, & alter pene Demosthenes.”⁵

The Wilton Shakespeare statue is almost identical to one in Westminster Abbey, also by Peter Sheemakers. Aside from their bases, the only difference between the statues is the verse inscribed on the scroll held by the statue:

Scroll in Wilton House

(from *Macbeth* 5.5.24-6)

LIFE's but a walking SHADOW
a poor PLAYER
That struts and frets his hour upon the
STAGE,
And then is heard no more!

Scroll in Westminster Abbey

(modified from *The Tempest* 4.1)

The Cloud cupt Tow'rs,
The Gorgeous Palaces
The Solemn Temples,
The Great Globe itself
Yea all which it Inherit,
Shall Dissolue;
And like the baseless Fabrick of a Vision
Leave not a wreck behind.

Table 1: Shakespeare Scroll Inscriptions at Wilton House and Westminster Abbey.

In Westminster Abbey, Shakespeare's finger points to the capitalized word “Temples,” whereas at Wilton House the it points to the all upper case word “SHADOW.” It may be preemptory to conclude that the word “SHADOW” means that William Shakspere was merely a shadow of the real author. The interpretation that Shakspere was a shadow player does not make sense because he was a real player. Perhaps there is some sense to be made of these words if the order is shifted to “shadow life stage player” since Shakspere is thought to have played parts such as Banquo's ghost. However, it seems unlikely that this would warrant special mention on the statue.

In times past it was considered intellectually piquant to place ciphers in full view, often as capital or italic letters within normal text. If this is the case then a full cipher problem may exist as “LIFE SHADOW PLAYER STAGE”. While this cipher appeared over a century after her death, Mary Sidney was deeply engaged in code, secret inks and advanced metaphysics of the day. For example, Walter Raleigh's half-brother Adrian Gilbert maintained an alchemist's laboratory at Wilton House.⁶

While a feasible solution to a cipher cannot be regarded as evidence, modern techniques that determine result log likelihoods can rank solutions in confidence. If

a solution is found it may be possible to incorporate this extra information into an improved Bayesian posterior probability for an authorship candidate. For example, if a cipher solution has a useful probability and the cipher credibly refers to Mary Sidney, then Bayes Rule may be used to calculate a significantly improved posterior probability for her authorship.

Methodology

The methodology of this research has two parts. The first part is a Bayesian estimation of authorship probabilities for each potential author based on ERGM analysis of the Elizabethan Social Network. The second phase of the research applies Bayesian methods to investigate an improvement in the probability of Mary Sidney's authorship given the log likelihood of a cryptographic solution to the Wilton House cipher.

1.1. ERGM Research Methodology

The general form of the ERGM model was first proposed by Frank & Strauss in 1986.⁷ Pairing, or dyad relationships, develop between two people (or nodes) based on the attractiveness of the attributes of each to the other. Triad relationships are triangles involving three people. Classic triad closure occurs when two nodes that have independent dyad relationships to a common node, form a relationship and thereby create a triangle.⁸ Stochastic transitivity is the process of increasing the number of triads through of evolving friendships, i.e., "the friends of my friends become my friends." This matches real dynamic social networks, in practice, which display a propensity for triad closure.

ERGM probabilistic models for the observed network of relationships are evaluated using logistic regression. A key advantage of the ERGM approach is that the restrictive assumption of dyadic independence may be relaxed in favor of stochastic transitivity. ERGM achieves this through a Geometrically Weighted Edge Shared Partner (GWESP) factor. This associates a higher probability with networks that have a greater density of triads. A scaling parameter of zero in GWSEP means that only the first shared partners are recognized. If a very high scaling parameter is used, then all triangles with any two of the three triad nodes are counted. However, geometric weighting decreases the marginal return from edge shared partners as the number of shared partners increases, thereby limiting triangle recognition by about threefold.

The probability of a connection between two people may be estimated by measuring the probability of the network forming, both without and with a connection between these two people. In social networking as in real life, the cliché "birds of a feather flock together" is apt.

When introducing a test relationship either no new triads may be formed, if there are no common partners in the network, or one or a number of new triads

may be formed. If the density of the network increases rapidly with a number of triangles being formed, then the probability of the network rises. This provides the attractive facility of ERGM analysis, which is to infer the probability of “that which is not there” given evidence that is present. It aims to naturally evolve the development of relationships to prospectively fill out “thin patches” in our understanding of social fabric, such as the enigmatic Shakespeare authorship question within the Elizabethan social network.

While probabilities for “that which is not there being there” illuminate investigation, such seer-like analysis must be kept in perspective: probabilities never substitute for tangible evidence.⁹ If an alternative answer to the Shakespeare authorship controversy is to be found, then it awaits the fortuitous discovery of definitive evidence.

The ERGM research in this paper is focused on an investigation of the Elizabethan social network to understand the network probability associated with a connection of each potential author to the Shakespearean works. The period under consideration is the golden era of Queen Elizabeth I (reigned 1558-1603) and James I (reigned 1603-1625). The scale of the task is not so daunting as London’s population in 1610 was only about 200,000.¹⁰ The London theater industry comprised about 1500 people, including playwrights, players and patrons.¹¹

The Elizabethan Social Network developed in this research shows that about 200 persons are prominent in the arts over the Elizabethan period. Data about the relationships between them, or nodes, are derived from many diverse and detailed sources, primarily available through the Internet. In addition to marital and filial relationships, the database captures friendships, favorites, patrons, reported affairs, disputes, denouncements, and associations such as Mary Sidney’s Wilton Circle of poets and Kings’ Men players and rumored organizations such as Walter Raleigh’s esoteric School of the Night. Subgraphs from the Elizabethan Social Network with a connection between each potential author and the First Folio are provided in Appendix: Social Network Connections of Potential Authors to the First Folio. Future development of the Elizabethan Social Network database may seek to expand the number of organizational entities.¹²

The nature of social network analysis means that results may only be interpreted in relation to the specific data used; relationship databases can never be complete, no matter how exhaustive the complication. There will always be many relationships that are personal, secret or did not warrant a mention in the records of the time. However, the 635 unique undirected positive relationships assembled in the Elizabethan Social Network are believed to capture the essence of the social fabric at the time. Negative relationships such as trenchant criticism and religious or political denouncements also exist, but have not been included in the ERGM analysis.

The positive relationships are pre-processed as adjacency matrix using Mathematica.¹³ This matrix is processed as a social network using the R-language *ergm* function of the *statnet* package.¹⁴ The *ergm* command to process a social network adjacency matrix file using a GWESP factor of 0.65, one million samples and 1,500 iterations is:

ergm (social network file name ~ edges + gwesp(0.65, fixed = TRUE), MCMCsamplesize = 1e+6, maxit = 1500, verbose = FALSE, control = control.ergm(steplength = 0.25), seed = 123,eval.loglik=TRUE)

The ERGM function returns the maximum log likelihood of the network. This represents the probability that a particular network occurs out of all possible networks.

Using the First Folio as a proxy for Shakespeare’s plays, the relative log likelihoods for link to each potential authorship candidate is calculated against the structure of social network relationships using the ERGM method. This provides the conditional probability of the network given the author $P(\hat{a} | \acute{a})$.

The conditional probability of an author given the network $P(\acute{a} | \hat{a})$ is the Bayesian interchange of $P(\hat{a} | \acute{a})$ and may be calculated with Bayes Rule:

$$P(\acute{a} | \hat{a}) = \frac{P(\hat{a} | \acute{a}) \times P(\acute{a})}{P(\hat{a})}$$

$$= \frac{P(\hat{a} | \acute{a}) \times P(\acute{a})}{P(\hat{a} | \acute{a}) \times P(\acute{a}) + P(\hat{a} | \bar{\acute{a}}) \times P(\bar{\acute{a}})}$$

where :

$P(\acute{a})$ = Prior probability of author

$P(\hat{a})$ = Total probability of network

$$= P(\hat{a} | \acute{a}) \times P(\acute{a}) + P(\hat{a} | \bar{\acute{a}}) \times P(\bar{\acute{a}})$$

$P(\hat{a} | \acute{a})$ = Probability of network conditional upon author connection to First Folio

$P(\hat{a} | \bar{\acute{a}})$ = Probability of Elizabethan Social Network (prior to any author connection)

$$\left\{ \begin{array}{l} P(\hat{a} | \acute{a}) \\ P(\hat{a} | \bar{\acute{a}}) \end{array} \right\} = e^{\alpha}, \text{ where } \alpha \text{ is the log likelihood of the applicable network}$$

The calculation requires an estimate of the probability of an author’s connection to the First Folio $P(\alpha)$ prior to the ERGM social network analysis. This may be calculated from the base Elizabethan Social Network log likelihood statistic γ for an edge taking into account the GWESP factor:

$$P(\alpha) = e^{\gamma}$$

where:

$$\gamma = a + b \times (c + d)$$

a = Edge log likelihood

b = GWESP log likelihood

c = number of nodes in common (est. 2)

d = number of edges that first enter triangles when the two nodes are joined (est. 1)

For comparison purposes the probability of each author's connection to the First Folio is assumed to be uniform across potential candidates.

1.2. Cryptographic Research Methodology

Cipher topics have a mixed heritage because of base rate fallacy such as the prosecutor's fallacy.¹⁵ Ciphers may have tens, hundreds or even thousands of possible solutions, so a single feasible solution provides no basis for causation.

There continues to be a fine balance between legitimate areas of academic research and the more degenerate aspects of cipher topics. Positive aspects have emerged in recent decades, which include the respected Bayesian sciences of probability, cryptography and search and marketing methods. Indeed, many people now enjoy the benefits of cipher research on a daily basis in activities from shopping and banking to email and spam detection. Another legitimate area for academic enquiry is a deeper understanding of the intricacies of history, philosophy, geopolitics and literature expressed through diplomatic intelligence methods. The second part of the research in this paper concerns a branch of literary history and diplomatic intelligence methods that has relevance to the social network research.

There are numerous techniques for enciphering a message. Two common methods are simple letter substitution ciphers and polyalphabetic substitution. The famous Caesar and Atbash ciphers are examples of simple letter substitution using code words and shifted or reversed alphabets. Vigenère ciphers are polyalphabetic versions of these that use multiply shifted code words. Encryption with Vigenère rotation can be near perfect if strict rules are followed during the encipherment. However, this is an exhaustive process. Shortcuts are usually taken that permit decipherment. For example, if a message is sufficiently long it is sometimes possible to detect a keyword using letter frequencies.

There are well-regarded programs for decipherment of Caesar and Vigenère ciphers.¹⁶ However the use of letter frequency analysis, together with a range of specific clue words, did not provide any optimism that the Wilton House "LIFE SHADOW PLAYER STAGE" message might be deciphered in this way.

Furthermore, the inability of such techniques is to be expected because of the exigency to achieve sensible words in both the message and its enciphered result. For this reason the cipher, if it is indeed one, is more likely to be a simple but clever mixed letter substitution. This assumption does not significantly simplify the task. The variation of spaces and perhaps some letter transposition create a huge solution space that can be daunting for well-regarded dictionary attack programs. The Decrypto package is used for mixed letter substitution decipherment in this research.¹⁷

The usual way to penetrate such ciphers is to constrain the solution space using a clue word drawn from human intuition about the context of the solution. While this may produce an interesting potential solution, it is analogous to a local rather than global minimum in optimization. There is no guarantee that the original message has been found.

For example, it may be observed that “shadow player” has two “a” letters separated by five unique letters with other unique letters before and after. Various clue words that match this pattern may then be pegged to the two “a” letters. While this pattern might seem fairly unusual, there are 322 surnames in the Welsh Medieval Database of Primarily Nobility and Gentry¹⁸ that match the pattern, for example Chamberlain, Golding and Oldcastle. “MarySidney” is also clue word based on the two “y” letters. Even “Elizabeth” is a surprisingly strong clue word as it directly matches two “e” letters and the “a.” However, none of these five clue words appear to generate a sensible result.

A number of criteria need to be satisfied for a clue word to produce a sensible result. The message is then subjected to a “dictionary attack” where the unmatched letters and spaces are varied to complement the clue word cipher letter equivalences. Progressively, some elegant words may be distilled that seem sensible with the clue word and the context. Sometimes, unusual words may indicate an interesting and desirable solution. For example, the Elizabethan spelling of “blood” as “bloud.” Finally, a table of trigram probabilities, which encompasses all the three-letter sequences in the recovered text, is applied to calculate a log likelihood for each candidate solution.

Following decipherment, the log likelihood of the solution may be used to adjust the probability of the relevant author. In Wilton House the relevant author is Mary Sidney, but could also be another candidate depending upon the nature of the deciphered message. The adjusted probability $P(\acute{a} \mid \grave{a} \cap \hat{a})$ of Mary Sidney’s authorship (\acute{a}) may be calculated with the general conditional version Bayes Rule, given the probabilities that Mary Sidney is the person referred to in the cipher (δ) and the cipher (β) with clue word is correct.

$$\begin{aligned} P(\acute{a} \mid \grave{a} \cap \hat{a}) &= \frac{P(\grave{a} \mid \acute{a} \cap \hat{a}) \times P(\acute{a} \mid \hat{a})}{P(\grave{a} \mid \hat{a})} \\ &= \frac{P(\grave{a} \mid \acute{a} \cap \hat{a})}{P(\grave{a} \mid \hat{a})} \times \frac{P(\acute{a} \mid \hat{a})P(\acute{a})}{P(\hat{a})} \\ &= \frac{P(\grave{a} \mid \acute{a} \cap \hat{a})}{P(\grave{a} \mid \hat{a})} \times \frac{P(\hat{a}) - P(\hat{a} \mid \bar{\acute{a}})P(\bar{\acute{a}})}{P(\hat{a})} \end{aligned}$$

where :

- $P(\acute{a})$ = Prior probability of Mary Sidney’s authorship
- $P(\hat{a})$ = $P(\hat{a} \mid \acute{a}) \times P(\acute{a}) + P(\hat{a} \mid \bar{\acute{a}}) \times P(\bar{\acute{a}})$
- = $e^{\hat{a}}$ (for cipher log likelihood \hat{a})
- $P(\bar{\acute{a}})$ = Probability that Mary Sidney is the Pembroke referred to in the cipher

As there is no information for the required distributions $P(\grave{a} \mid \acute{a} \cap \hat{a})$, $P(\grave{a} \mid \hat{a})$ and $P(\hat{a} \mid \bar{\acute{a}})$, the adjusted (posterior) probability $P(\acute{a} \mid \grave{a} \cap \hat{a})$ of Mary Sidney’s authorship is calculated using Monte Carlo

simulation. The unknown distributions are simulated by Bates Distributions parameterized by the minimum and maximum expected values in each case. The Bates Distribution can represent uniform, triangular and quasi-normal distributions that range from a minimum to maximum value. The simulation outcome for Mary Sidney's authorship probability is found to be similar for each distribution type.

2. Results

2.1. ERGM Results

The maximum log likelihood of the Elizabethan Social Network, excluding all potential author connections, rises from the dyad only value of about -1588 to about -1576 as the Geometric Weighted Edge Shared Partner (GWESP) weighting parameter α increases from 0 to 0.7. Illustration 1 provides a curve fitted to the results that suggests only minor improvements in log likelihood might be possible by increasing α beyond 0.7.

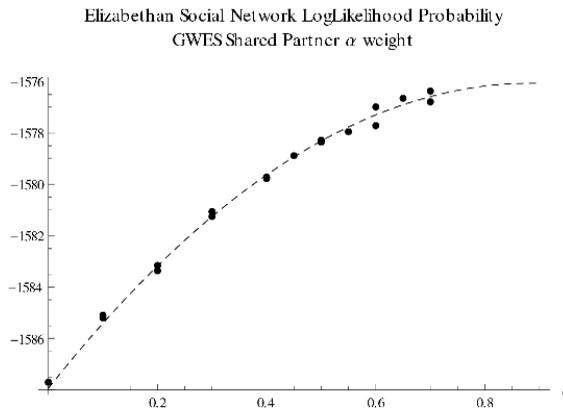


Illustration 1: Elizabethan Social Network loglikelihood as a function of the GWESP factor.

Inherently, the Elizabethan Social Network is incipiently unstable, which is common for sparse social models. Ideally, the index of instability would be less than unity but could range up to infinity. The index of instability for the Elizabethan Social Network remains at about 3 with α ranging from zero to 0.65. With an increase in α to 0.7 the network becomes sharply degenerate resulting in very long search times that mostly achieve no increase in log likelihood. An optimal α is therefore a trade-off between improving model log likelihood and model stability. In addition, the reliable estimation of the edge and GWESP factors needs to be verified using Monte Carlo Markov Chain (MCMC) diagnostics. Taking these criteria into account, the maximum reliable α for the Elizabethan Social Network is found to be 0.65.

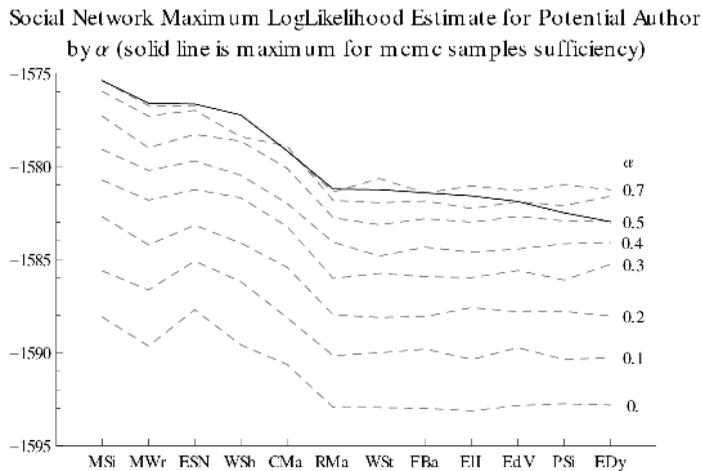


Illustration 2: Loglikelihood of Elizabethan Social Network with Author connection to First Folio as a function of GWESP factor.

Illustration 2 provides the maximum reliable log likelihood of the potential author models (solid line) where the GWESP α parameter is manually tuned for simultaneous incipient degeneracy and reliable parameter estimation. This maximum reliable profile is overlaid on dashed iso- α profiles. It may be noted that networks with higher log likelihoods tend to be reliable at the higher α parameters of 0.65 to 0.7, while the α decreases toward 0.5 for lower likelihood author profiles. Notwithstanding this reliability effect, the maximum reliable log likelihoods profile is consistent with the overall structure of the iso- α profiles.

In Illustration 2 and subsequent illustrations categorized by author, the Elizabethan Social Network (ESN) and the authorship candidates and are referred to by three-letter labels based on their initials: William Shakespeare (WSh), Edward de Vere (EdV), Christopher Marlowe (CMa), Francis Bacon (FBa), Mary Sidney (MSi), Philip Sidney (PSi), Roger Manners (RMa), William Stanley (WSt), Edward Dyer (EDy), Elizabeth I (EII) and Mary Wroth (MWr).

The log likelihood of two models of potential authors, Mary Sidney (MSi) and Mary Wroth (MWr), exceeds that of the Elizabethan Social Network (ESN) prior to any author connections. The log likelihood for the William Shakespeare model is about the same as that of the Elizabethan Social Network. All other potential authorship models significantly impair the log likelihood of the network, suggesting that these models are less likely than the Elizabethan Social Network itself.

Bayes Factors may be used to rank preferences for authorship candidates. In Table 2 (below), cell values express the strength of preference for a column author over a row author as *Decisive*, *Very Strong*, *Strong*, *Substantial* and *Barely* a preference.

Strength of Bayes Factors for each column model compared to the row models: Decisive(4), Very Strong(3), Strong(2), Substantial(1), Barely (~)											
	MSi	MWr	ESN	WSh	CMa	RMa	WSt	FBa	ELI	EdV	PSi
MWr	1										
ESN	1	~									
WSh	1	~	~								
CMa	3	2	2	1							
RMa	4	4	3	3	1						
WSt	4	4	3	3	1	~					
FBa	4	4	4	3	1	~	~				
ELI	4	4	4	3	2	~	~	~			
EdV	4	4	4	4	2	~	~	~	~		
PSi	4	4	4	4	2	1	1	~	~	~	
Edy	4	4	4	4	3	1	1	1	1	~	~

Table 2: Bayes Factor Rankings of Authorship Probability.

The first column in Table 2 suggests a *Substantial* preference for Mary Sidney (MSi) over Mary Wroth (MWr), William Shakespeare (WSh) and the Elizabethan Social Network (ESN) without any other authorship candidate. There is a *Very Strong* preference for Mary Sidney over Christopher Marlowe (CMa) and a *Decisive* preference for Mary Sidney over all other candidates.

The second column suggests that the Elizabethan Social Network (ESN) without any authorship candidate is *Barely* preferred to William Shakespeare (WSh), *Strongly* preferred to Christopher Marlowe (CMa) and *Very Strongly* or *Decisively* preferred to all other candidates.

The very poor Bayes rankings of Roger Manners (RMa), William Stanley (WSt), Francis Bacon (FBa), Elizabeth I (ELI), Edward de Vere (EdV) and Philip Sidney (PSi) suggest that these are inferior authorship candidates. Within this group of potential authorship candidates the relative preferences are not meaningful.

While the analysis above clearly establishes the preference for potential authors, the conditional probability of each author may be calculated with Bayes Rule. It is necessary to estimate the prior probability $P(\acute{a})$ that a potential author has a connection to the First Folio. This may be calculated from the log likelihood $\tilde{\alpha}$ of the Elizabethan Social Network, given the edge log likelihood and GWESP factor.

The prior probability assumptions suggest a value for $P(\alpha)$ of approximately 0.21. This appears to be a reasonable estimate given the limited number of potential candidates and that literature and history research suggests that these candidates are significantly preferred to other people in the network. This prior probability might also be considered conservative given the potential for unknown, unclosed triads in the vicinity of the true author that are currently not represented in the Elizabethan Social Network because they are not yet known or perhaps forever secret.

As shown in Illustration 3 (below), the effect of increasing the prior probability from 0.05 to 0.25 is to amplify the posterior probability of the most likely candidates. As relative author probabilities might be considered more important than an exact value of an author probability, a midpoint prior probability of 0.17 is

considered satisfactory for comparative purposes.

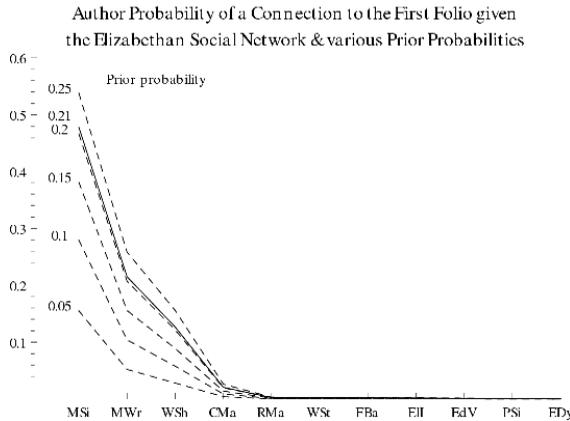


Illustration 3: Author Probability based on ERGM analysis of Elizabethan Social Network

Based on a prior probability $P(\hat{a})$ of 0.21, Mary Sidney has a conditional authorship probability $P(\hat{a} | \hat{a})$ of 0.48. The second most likely candidate, Mary Wroth, has a conditional probability of 0.22 (which is 45% that of Mary Sidney). The conditional probabilities for the two next most likely candidates, William Shakespeare and Christopher Marlowe, are 0.13 and 0.02 (26% and 4% that of Mary Sidney) respectively.

Introducing a test connection between various potential authors and the First Folio may lead to the formation of three new triads. Illustration 4 shows that three new triads arise from connecting Mary Sidney Countess of Pembroke with the First Folio, two in connecting each of Mary Wroth and William Shakespeare, one in connecting Christopher Marlowe and none in connecting the remaining potential authors.

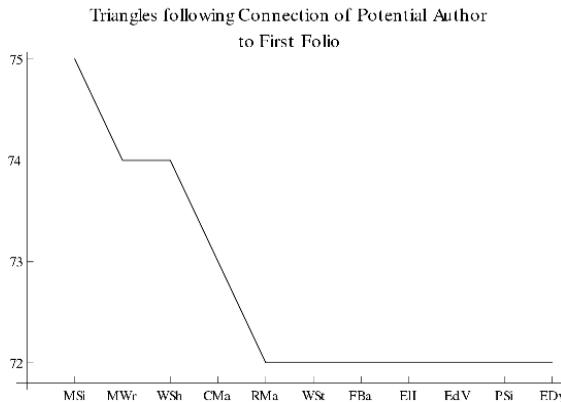


Illustration 4: Increase in Triads (triangles) resulting from a Relationship between the Author and the First Folio.

This suggests that particular authorship probabilities may be improved through intensifying the investigation of potential triangular relationships surrounding the author.

2.2 Cryptographic Analysis Results

Of the many clue words tested, “pembroke” provides consistent solutions enhanced by the unusual word “bloud” and with log likelihoods that are significantly greater than those for other keywords. Table 3 provides the best four solutions for the clue word “pembroke.”

Candidate Solution	Effective letter arrangement of “life shadow player stage”	Log likelihood \bar{a}
I who have pembroke bloud	W PLA LIFE YERSTAGE SHADO	-1.895
In a few no pembroke bloud	W LIFE PLAYER STAGE SHADO	-1.965
A two bloud pembroke wife	W PLA SHADO YERSTAGE LIFE	-2.121
Two wives pembroke bloud	PLA LIFEW YERSTAGE SHADO	-2.274

Table 3: Potential Solutions of the Wilton House Shakespeare Cipher.

The highest probability cipher solution is “I who have Pembroke bloud.” However, this is not straightforward as the word “life” needs to be relocated into the middle of the word “player.”

As the first and second Earls of Pembroke were men of action rather than letters, it is unlikely that this cipher solution refers to their Pembroke blood. Nor is Mary Sidney directly of Pembroke blood, although her epitaph (by Ben Johnson, William Browne or perhaps her son William “Sidney’s sister and Pembroke’s mother”) might be seen to satisfy the cipher result from an unusual perspective. Setting aside this interpretation, the cipher result suggests further research in the collateral branches of the Herbert family might be worthwhile. The collateral branches are descended from the first and second creations of the first Earl and had given rise to more than 300 male Herberts by Elizabethan times. Collateral branch analysis is somewhat complicated by traditional family names. For example, 21% of male Herberts are named William, and a further 40% are John, Thomas, Richard or Edward.

The second cipher result “In a few no Pembroke bloud” is quite elegant because it has no letter rearrangement and only a single letter rotated from the end of the cipher to the beginning. The statement seems to imply that the work of a Pembroke can be found in the majority of Shakespeare’s plays. In this context the word “bloud” may be less of a biological imperative than in the first cipher result and more like a metaphor for Pembroke workmanship. It might be quite logical to argue that Pembroke workmanship is the vital force in Shakespeare’s plays. This Pembroke

work may well, of course, be that of the Countess of Pembroke, Mary Sidney, who has been highlighted by the social network analysis.

The third cipher result is “A two blood pembroke wife.” Mary Sidney’s bloodlines could well justify such a claim arising from the Dudleys, Northumberlands, Mortimers and Nevilles.¹⁹ It might be recalled that the affinity between the Dudleys, Sidneys and Herberts was exceedingly strong.²⁰ For example, in his defense of the Earl of Leicester (1584), Mary’s brother Phillip wrote “I am a Dudley in blood, that Duke’s daughters son; and do acknowledge, - though, in all truth, I may justly affirm that I am, by my father’s side, of ancient and always well-esteemed and well-matched gentry, - yet I do acknowledge, I say, that my chieftest honour is to be a Dudley.” There is an interesting suggestion that Mary Sidney proclaim her blood in the 1608 play *Coriolanus* (Folio 1, 1623, 1.9 lines 763-64): “My Mother, who ha’s a Charter to extoll her Blood, ...”

Elizabethans were fascinated by notions of blood. The poet John Donne was a member of the illustrious First Friday Club of poets and playwrights that met regularly at the Mermaid Tavern. His analogy of God’s harmony to the two bloods of Philip and Mary Sidney would provide compelling ambience for any inscription. In Donne’s well-known eulogy *Upon the Translation of the Psalms by Sir Philip Sidney and the Countess of Pembroke*, he describes the fusing of Philip and Mary’s blood with God’s Spirit: “ETERNAL God - for whom who ever dare / Seek new expressions, do the circle square / ... That, as thy blessed Spirit fell upon / These Psalms’ first author in a cloven tongue / ... So thou hast cleft that Spirit, to perform / That work again, and shed it here, upon / Two, by their bloods, and by Thy Spirit one; /A brother and a sister, made by Thee / The organ, where Thou art the harmony.”

Another interpretation of this third cipher result is Mary Sidney’s dual personas. The first is her public profile of a quiet, private, pious wife who translates Psalms. The other is her covert but presumably boisterous life of Court masques and Shakespearean productions. The latter is certainly supported by Mary’s sponsorship of the Wilton Group of poets, her hosting the presentation of plays by Pembroke’s Men (*Titus Andronicus*, *The Taming of the Shrew* and *Henry VI Part 3*) and premiering of plays to Elizabeth I at Wilton House (*As You Like It*).

The final cipher result, “two wives Pembroke blood,” suggests that two Pembroke wives have collaborated. The social network analysis highlighted Mary Wroth. Although she was not by law a Pembroke wife, following the death of her husband in 1614 when she was 27 years old, Mary Wroth became the mistress of Mary Sidney’s first son, William 3rd Earl of Pembroke, and mother of his only two children that did not die at birth, albeit they remained illegitimate.

Another Pembroke wife candidate might be Magdalen Newport (c.1570-1627), who married Richard Herbert (c.1550-1596) of Montgomery Castle (about 150 miles from Wilton). Magdalen was Sir Phillip Sidney’s childhood friend and outwardly shared piety with Mary Sidney. John Donne saw both Magdalen and Mary Sidney as similarly gifted and praised them both. He dedicated the *Holy Sonnets* to Magdalen and presented the sermon at her funeral. Magdalen’s children were highly regarded in the arts. Her first son, Lord Edward Herbert of Chirbury

(1582-1629), was a noted philosopher and historian. Mary Sidney's son William 3rd Earl of Pembroke also assisted Magdalen's fifth son, George (1593-1633), who was Cambridge University's Public Orator and a celebrated poet to whom Francis Bacon dedicated his *Translation of Certaine Psalmes*.

Using the general conditional version Bayes' Rule with a background event, the posterior probability $P(\hat{a} \mid \bar{a} \cap \hat{a})$ of Mary Sidney's authorship \hat{a} given the probabilities that Mary Sidney is the person referred to in the cipher \bar{a} and the cipher \hat{a} is correct with clue word "pembroke" may be calculated as.

$$\begin{aligned}
 P(\hat{a} \mid \bar{a} \cap \hat{a}) &= \frac{P(\bar{a} \mid \hat{a} \cap \bar{a})}{P(\bar{a} \mid \hat{a})} \times \frac{P(\hat{a}) - P(\hat{a} \mid \bar{a})P(\bar{a})}{P(\hat{a})} \\
 &= \frac{P(\bar{a} \mid \hat{a} \cap \bar{a})}{P(\bar{a} \mid \hat{a})} \times \frac{0.1503 - P(\hat{a} \mid \bar{a})(1 - 0.42)}{0.1503}
 \end{aligned}$$

where :

- $P(\hat{a})$ = Prior probability of Mary Sidney's authorship (0.42)
- $P(\bar{a})$ = $e^{-1.895}$ (for the best cipher log likelihood $\bar{a} = -1.895$)
= 0.1503
- $P(\hat{a})$ = Probability that Mary Sidney is the Pembroke referred to in the cipher

The posterior probability $P(\hat{a} \mid \bar{a} \cap \hat{a})$ of Mary Sidney's authorship is estimated with Monte Carlo simulation using the following three random variates:

- $P(\bar{a} \mid \hat{a} \cap \bar{a})$ = random variate 6 Bates Distribution[3] with high probability over {0.9,1.0}
- $P(\hat{a} \mid \bar{a})$ = random variate 6 Bates Distribution[3] with medium probability over {0.65,1.0}
- $P(\bar{a} \mid \bar{a})$ = random variate 6 Bates Distribution[3] with low probability over {0.05,0.3}

The result is an estimate of the mean posterior probability $P(\hat{a} \mid \bar{a} \cap \hat{a})$ of Mary Sidney's authorship of 0.46. Given the standard deviation of 0.17, this is a non-material change in the prior probability of 0.48 from the social network research. The main reason for this finding is that there is a non-trivial probability of the cipher existing $P(\bar{a} \mid \bar{a})$ in the absence of Mary Sidney's authorship. For example, another Pembroke or another person entirely could well be the author.

The posterior probability $P(\hat{a} \mid \bar{a} \cap \hat{a})$ of Mary Sidney's authorship would have increased significantly if the cipher had provided unambiguous and credible evidence of her authorship. For example, the presence of her name in the cipher would have reduced the probability of the cipher existing $P(\bar{a} \mid \bar{a})$ in the absence of her authorship to almost zero, thereby increasing the posterior probability of Mary Sidney's authorship $P(\hat{a} \mid \bar{a} \cap \hat{a})$ towards one (i.e., 100% probability).

As interesting as this cipher is in providing directions for further investigation, no reliance can be placed on it for the purposes of enhancing Mary Sidney's probability of authorship. Mary Sidney's probability of authorship therefore remains unchanged from the social network analysis estimate of 0.48.

3. Discussion

This research does not set out to unequivocally answer the authorship question because social network analysis and other Bayesian inference can only provide insights. However, some of these insights are of interest.

The ERGM social network research suggests that Mary Sidney is the preferred authorship candidate with an authorship probability of 0.48. A relationship between Mary Sidney and the First Folio materially improves the log likelihood of the Elizabethan Social Network. A cryptographic analysis of the Wilton House Shakespeare statue scroll found interesting solutions to the cipher but these were insufficient to enhance the probability of Mary Sidney's authorship. However, these cipher solutions may provide directions for further research in this topic.

The second most likely candidate, Mary Wroth, has a probability of authorship of 0.22. A relationship between Mary Wroth and the First Folio provides a marginally positive increment over the probability of the Elizabethan Social Network, which is 0.21.

William Shakspeare remains enigmatic as ever due to the limited direct and indirect evidence of his relationships. His authorship probability is 0.13, which is only 26% of that of Mary Sidney and significantly less likely than the base Elizabethan Social Network. What little is known of his social network is sufficient neither to discount him as a potential author nor to favor him. Interestingly, the prospects for his authorship are enhanced by the outcome that his prospective authorship does not reduce the log likelihood of the network from that of the Elizabethan Social Network. It might be conjectured that William Shakspeare's probability of authorship is finely balanced and poised to increase with additional information.

In sharp contrast, the potential authorship of the other candidates (Christopher Marlowe, Roger Manners, William Stanley, Francis Bacon, Elizabeth I, Edward de Vere and Philip Sidney) has negligible probability. There may be valid reasons for this. For example, Christopher Marlowe and Philip Sidney have exceptional literary reputations. Their low authorship probabilities may be attributable to their lengthy overseas sojourns and tragic early deaths in 1593 and 1586 respectively. Therefore, each had limited opportunities to establish the social relationships necessary to enhance their network probability.

Perhaps a surprising outcome is Edward de Vere's low authorship probability and that a relationship between Edward and the First Folio materially reduces the log likelihood of the Elizabethan Social Network. The reason for this appears to be that while Edward de Vere could be described as a supernova of Elizabethan literature patronage, his influence appears to have remained quite distinct from that of the Dudley, Sidney and Herbert family cluster with whom he appears to have had little empathy.

For example, there is a story that Edward de Vere insulted Philip Sidney at a tennis game and threatened to kill him.²¹ Although each of Edward de Vere and the Herberts showed great deference to William Cecil 1st Baron Burghley, both seemed cool to Cecil's efforts to marry Mary Sidney's first son, William, and Edward de Vere's 13-year-old daughter, Bridget. Conveniently the issue of dowry payment timing frustrated the plan. Relationships between Edward de Vere and Mary Sidney appeared to have remained thorny, with no further social or commercial relationships developing notwithstanding their arts patronage. Shortly after Edward's death, his daughter Susan and Mary Sidney's second son Phillip announced their plans to marry. Although Susan's uncle Robert Cecil intervened against the arrangement, James I overrode his objection to approve the union and even seems to have romped in the nuptial bed.²²

The Sidneys' rich contribution to the English Renaissance has been recognized: "We remember how much the Florentine Renaissance owed to the Medici, but we forget that a similar debt was owed by the English Renaissance to the Sidneys."²³ The underlying reason appears to be Elizabeth's deep-seated suspicion of potential claimants to the throne, which led to her rather overt discrimination against Mary Sidney's brothers, Philip and Robert. Elizabeth's dislike of Philip Sidney is evident in her often repeated statement about Robert Devereux, 2nd Earl of Essex: "We shall have him knocked o' the head like that rash fellow Sidney."²⁴ Following Elizabeth I's death in 1603, James I promptly atoned for Elizabeth I's sustained dissonance by elevating Mary Sidney's brother Robert and sons William and Philip Herbert to positions of substantial wealth and power.

Some of Shakespeare's plays had the potential to be interpreted as seditious. For example, Elizabeth I said of the performance of *Richard II* at the Globe on the eve of the 1601 Essex rebellion, "I am Richard II, know ye not that."²⁵ The title page of the 1598 reprint is the first of Shakespeare's plays to actually state that it was by William Shakespeare. Although the players' manager and one of the actors were arrested and gave evidence, William Shakespeare was neither arrested nor mentioned as the author of the play in Court records.

If Mary or Philip Sidney's literature had been interpreted as seditious or treasonous, then their estates and perhaps their lives would have been forfeited, as had been the case with their grandfather John Dudley Duke of Northumberland. It is notable that Mary Sidney's husband, Henry Herbert 2nd Earl of Pembroke, who died in January 1601, ingeniously ameliorated this pervasive threat to the family's wealth by disenfranchising Mary of the corpus of family wealth, even the traditional widow's one-third and her personal jewelery, whilst otherwise arranging generous income for her and access to assets through contracts to manage the family estates.

Although the ERGM social network analysis did not highlight Philip Sydney as a prime authorship candidate, it is apparent that his literary brilliance was shared with his sister Mary Sidney. Philip Sidney's sole works, the pastoral love story *The Countess of Pembroke's Arcadia* and sonnets *Astrophel and Stella*, might be considered impressive in their classical scope and workmanship but otherwise rather uninteresting. Sidney's women are objects of puppy love, lacking the strength of

character that distinguishes Shakespeare's women.²⁶

In *A defence of Poesie and Poems* (1581), while generally dismissing English drama as a whole, Philip Sidney gives only limited approval to the first major use of blank verse in Thomas Sackville and Thomas Norton's *Gorboduc* (1561). Sidney writes: "Our tragedies and comedies, not without cause, are cried out against, observing rules neither of honest civility nor skilful poetry. Except *Gorboduc* ... as it is full of stately speeches, and well-sounding phrases, climbing to the height of Seneca in his style ... yet in truth, it is very defectuous in the circumstances, which grieves me, ... for it is faulty both in place and time. But if it be so in *Gorboduc*, how much more in all the rest?" Details of time and place never seemed to concern Shakespeare. Shortly after Philip Sidney wrote these words, Christopher Marlowe developed blank verse into what Ben Jonson described as Marlowe's "mighty line" and Shakespeare ever more cleverly exploited the art.

Mary Sidney's highly admired completion of her brother Philip's *Sidney Psalms* was presented to Elizabeth I in 1599. Its inspired and vivid translations led to immediate acclaim. In major part the work was that of Mary, suggesting that she may have possessed the greater talent. Writing in 1611, Aemilia Lanyer considered Mary the sole translator.²⁷

However, a year before Mary Sidney presented the *Sidney Psalms*, Meres wrote in *Palladis Tamia* (1598) that in Mary's patronage of the arts she "is very liberal unto Poets." He compared her to Octavia Minor, who generously rewarded Virgil for each verse of the Aeneid he recited. Just as Octavia was devoted to her brother Augustus, the first Roman Emperor, so was Mary to her brother Philip. Indeed, Philip Sidney is said to have similarly rewarded Edmund Spenser for every stanza of the Shepherd's Calendar that he recited.

Meres also described Mary as a "most delicate poet" and likened her to Sappho (c. 600 BCE), for which there are a number of parallels. For example, both were poets of love, careful to remain at length from politics, active developers of verse structure, leaders of literary circles and the single female poet among highly regarded male poets.²⁸

Although much of Sappho's work is lost, it is clear from the surviving epigrams that she was a Mysteries lyricist. For example, in one of the earliest references to Sappho, Dioscórides (c. 250 BCE) praises her while referring to the rites of Persephone and of Adonis: "O Sappho, sweetest support for young passions, / You must surely be keeping company with the Muses, / Honoured by ivied Helicon and by Pieria, / for the songs of the Muse from Eresus equal theirs / or else it's the God of weddings, Hymen, / who stands by you over the bridal bed, torch in hand; / or else you share Aphrodite's weeping for young Adonis, / and so come to see the holy grove of the blessed. / Greetings wherever you are, lady, greetings as to a god: / for your songs, your immortal daughters, are with us still."²⁹ Aside from the context of the Mysteries, Mary's love for her dead brother Philip is plainly analogous to Aphrodite weeping for young Adonis.

It became popular to refer to Sappho as the tenth muse, fourth Grace or second Helen. For example, Antipater of Sidon (c. 150 BCE) writes: "Hearing the

songs of honey-voiced Sappho, the goddess Memory stood amazed, / [Mother of nine immortal muses], she wondered: / could there, on earth, be a tenth?"³⁰ (Antipater also laments for Sappho: "O ye Fates twirling the / triple thread on the spindle, why spun ye not an / everlasting life for the singer who devised the / deathless gifts of the Muses of Helicon?"³¹ In likening Mary Sidney to Sappho, Meres really compares Mary to a Mysteries lyricist and to the person who coordinated or brought the Muses together through a circle of playwrights. He directly compares her neither to the Muse of sacred poetry and hymns, Polyhymnia, which would have been appropriate if Meres had knowledge in advance of the *Sidney Psalms*, nor to the other Muses that inspired Shakespeare's themes: Calliope (heroic poetry), Melpomene (tragedy) and Thalia (comedy).

It may be appropriate to sever Mary Sidney's work in these themes from her quite different efforts in theater patronage and entertainment. The latter is consistent with suggestions from the cryptographic research. Mary Sidney's role as patron, sponsor, motivator, circle leader, coordinator and project manager is very influential and important, but is distinct from direct authorship of tragedies, histories and comedies.

The proposition that Shakespeare's plays were project managed by Mary Sidney is also supported by hesitant authorship attributions. The first three of the initial eight plays (*Titus Andronicus* [1594], *Taming of the Shrew* and *Henry VI Part 3* [1595]) substituted statements that various acting companies had performed the plays for authorship attributions. For example, the title page of *Titus Andronicus* unusually stated that it had been acted by all three of the Pembroke's Men, Derby's Men, and Sussex's Men. The title page of *Henry VI Part 3* named only the Pembroke's Men.

The first play to mention the name William Shakespeare was *Loves Labour's Lost* (1598), which was the ninth play to be published. It recorded him not as author but merely noted that William Shakespeare corrected and augmented the plays. Even later, within a year two of the first eight anonymous plays being reprinted, it was merely noted that William Shakespeare had "augmented" one and "corrected" another. The third of the anonymous plays to be reprinted, *Richard II* (1598 reprint) was the first play to state explicitly that it was "by William Shakespeare." Incongruously, the printing of *Henry V* (1600) again reverts to anonymous authorship. The next ten plays contiguously stated his authorship.

If Mary Sidney's primary roles were primarily those of patron, sponsor, motivator, coordinator and project manager, then she has very strong associations with the plays. It is also feasible that she developed strong female characters in the plays, such as Portia in *The Merchant of Venice*.³² Recalling Mary's enthusiastic participation in Ben Jonson's Court masques, Mary may have been an original contributor to masque-like scenes in many plays.

It is notable that Mary Sidney's niece Mary Wroth developed almost as many social connections that favor potential authorship links to the First Folio as did her aunt. Perhaps this is unsurprising given Mary Wroth's literary talents, her father's patronage of the arts in London, the extensive sojourns with Mary Sidney at Wilton

House from childhood onwards, and that she also participated enthusiastically in Court masques designed by Ben Jonson and Inigo Jones. The cryptographic phase of this research provides little positive support for Mary Wroth's involvement in the plays. Perhaps the most that can be advanced in support of Mary Wroth authorship is that she, in the same way as Mary Sidney, may have helped produce or project manage the plays and have contributed to the masque-like elements in Shakespeare's later plays.

From an evaluation of the social network results in the context of the period, it may be the case that First Folio is as likely to be a tribute to the life of Mary Sidney as it is to William Shakespeare. Although Mary Sidney has one epitaph ascribed to her friend Ben Jonson, it is suspected that he didn't write it. This hardly constitutes the usual outpouring of grief that accompanies the passing of a highly regarded poet or playwright. Nor is absence of grief consistent with the torchlit procession of over one hundred coaches that attended Mary's body from London to Salisbury Cathedral for burial, where there is no monument to Mary Sidney.³³

Prior to commencing a large and expensive project, publishers would often seek an indication of buyer demand by pre-announcement of the publication in a book fair catalogue. The concept of a folio of Shakespeare plays was commercially announced in 1622, in a semiannual Frankfurt Buch Mess Katalog. This was within about six months of Mary's death. The printed First Folio subsequently became available in December 1623.

4. Conclusion

Exponential Random Graph Model (ERGM) analysis of the Elizabethan Social Network facilitates an assessment of potential authorship connections to the First Folio. The analysis provides some support for the authorship of Mary Sidney Countess of Pembroke, her niece Mary Wroth and is ambivalent in regard to William Shakspeare.

Mary Sidney and her niece Mary Wroth have the potential to form multiple relationships with the First Folio, which increases the log likelihood of the Elizabethan Social Network and increases their probability of authorship or involvement in the plays. Decipherment of an inscription on a statue of Shakespeare at Wilton House provides interesting context and directions for future research, but does not enhance Mary Sidney's probability of authorship.

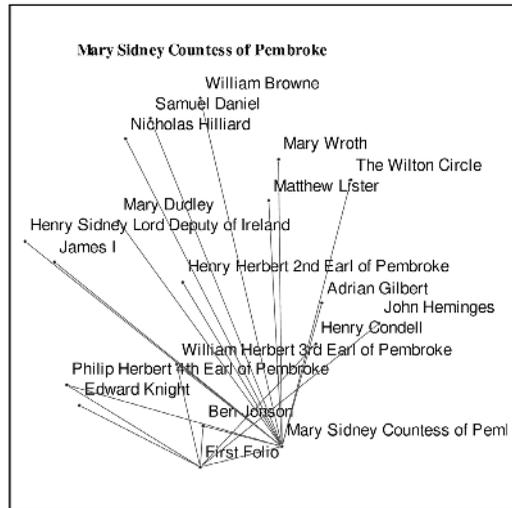
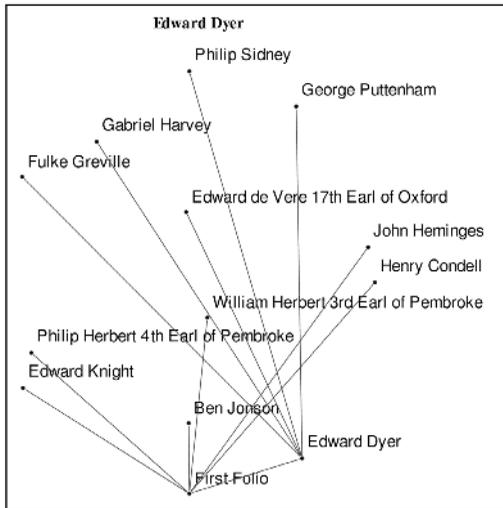
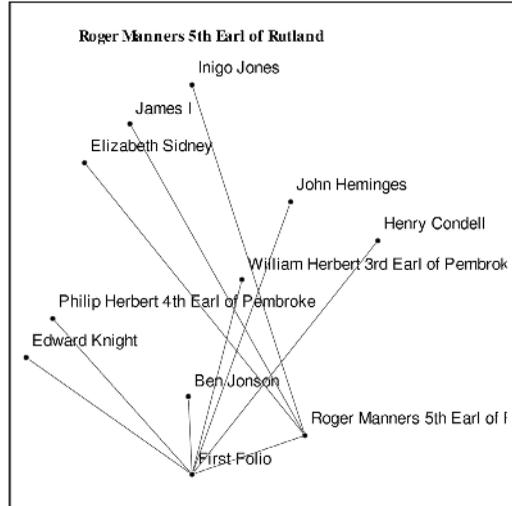
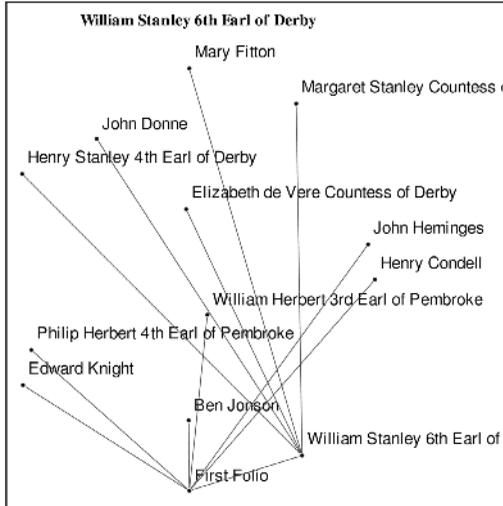
William Shakspeare's authorship remains enigmatic due to the limited extant information about his social relationships. This contributes to a low probability of authorship in social network research. While William Shakspeare's authorship does not increase the log likelihood of the Elizabethan Social Network, it is significant that neither does his authorship reduce it.

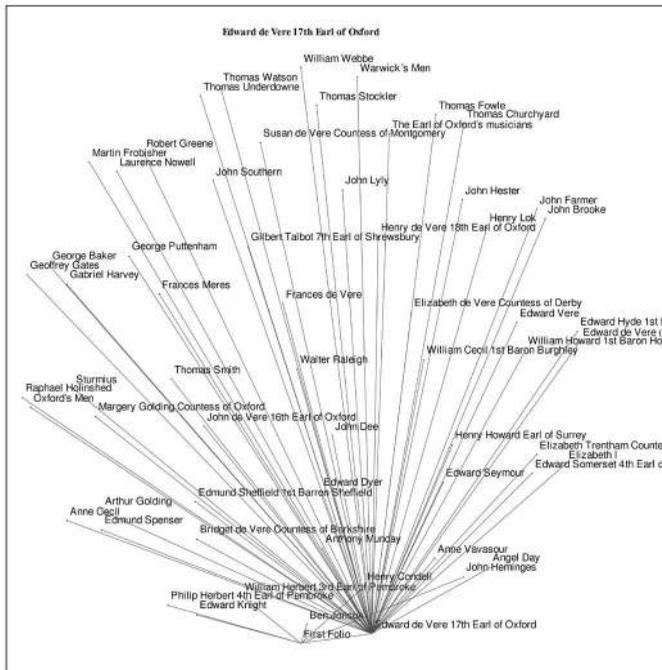
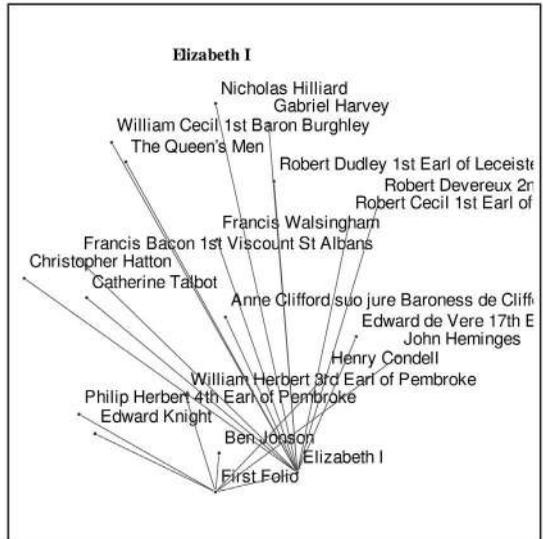
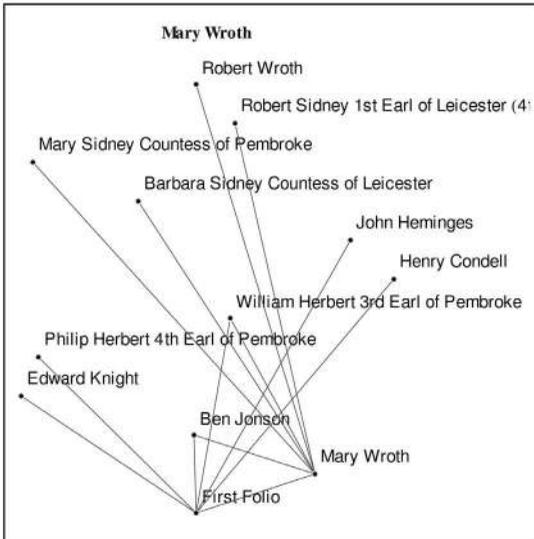
Perhaps the most significant finding is that the log likelihood of the Elizabethan Social Network is materially reduced by the potential authorship of Christopher Marlowe, Roger Manners, William Stanley, Francis Bacon, Elizabeth I,

Edward de Vere and Philip Sidney.

Conclusions for all Bayesian probability studies are limited to the data and may not be generalized. Enhancement of the probability of any potential author relationship with the First Folio is possible with the discovery of additional dyad and triad social network relationships.

Appendix One: Social Network Connections of Potential Authors to the First Folio





Endnotes

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